Fermat Comes to America: Harry Schultz Vandiver and FLT (1914–1963)

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Fermat's Last Theorem (FLT) was long known as the most famous of all unsolved mathematical problems. Familiar even to laymen, it attracted the attention of both professional mathematicians and amateurs, many of whom thought they held the key to its solution. Ferdinand Lindemann, who was suddenly vaulted to fame when he proved the transcendence of π in 1882, tried to solve FLT several times over the remainder of his career, only to come up with one faulty proof after another. Nor was he alone in this regard; indeed, FLT stood in a league by itself when it came to the number of incorrect proofs that found their way into print. As for failed attempts by amateurs who sent their "solutions" to mathematicians all over the world but which (thankfully) remained unpublished, the number cannot even be estimated.

These circumstances make it easy to understand the general excitement that surrounded this story when Andrew Wiles finally completed his general proof of Fermat's conjecture in 1994. As news of this impressive achievement rippled through the mathematical community, it also made headlines that attracted the attention of broad lay audiences who had no chance of grasping the ideas behind his work. Public interest in Wiles's personal story and his long quest to solve FLT added an unusual sense of drama to the accomplishment itself. His curiosity about the problem from childhood, his eight years of self-imposed seclusion that led to the breakthrough, the tension that followed discovery of a non-trivial mistake in his initial proof, and the final resolution eight months later in collaboration with Richard Taylor, all these elements only enhanced interest in this appealing story.

Still, from a broader perspective, informed individuals might well shake their heads when reading the overly dramatized popular versions of the quest to solve FLT (a tendency occasionally found even in accounts written for more professional audiences). The most striking example of this is surely Simon Singh's best-selling book, *Fermat's Enigma*, a book that conveyed to very broad audiences the excitement human beings feel about doing mathematics. Its readers learn that Fermat's conjecture "tormented lives" and "obsessed minds" for over three centuries, and thus constituted "one of the greatest stories imaginable." On the front flap of some editions one reads that FLT became the Holy Grail of mathematics and that Euler "had to admit defeat" in his attempts to find a proof, while:

> Whole and colorful lives were devoted, and even sacrified, to finding a proof. . . . Sophie Germain took on the identity of a man to do research in a field forbidden to females. . . . The dashing Evariste Galois scribbled down the results of his research deep into the night before venturing out to die in a duel in

Figure 1. Harry Schultz Vandiver and son, Frank, ca. 1930 (HSV).

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1832. Yutaka Taniyama . . . tragically killed himself in 1958. Paul Wolfskehl, a famous German industrialist, claimed Fermat had saved him from suicide.

On opening that book, one reads that "The Last Theorem is at the heart of an intriguing saga of courage, skullduggery, cunning, and tragedy, involving all the greatest heroes of mathematics."

Not surprisingly, a closer and more sober examination of the actual historical evidence surrounding research on Fermat's problem brings to light a far less dramatic version of these events. This is not to say that the history of FLT lacks interest, but the story hardly warrants the sense of high drama that recent writers have brought to it.[1] None of the mathematicians who appear in Singh's account (except for Wiles himself) ever devoted *sustained* research efforts purely focused on an attempt to solve this famous problem. In fact, many of those mentioned in his book showed only the slightest interest in solving it, whereas only one mathematician prior to Wiles took a similar passionate lifelong interest in FLT. This was Harry Schultz Vandiver (1882–1973), a figure who does not even appear in Singh's book and who is only mentioned marginally in most other recent accounts.

Vandiver devoted nearly all of his professional life to resolving this famous problem, a quest that set him apart from his fellow mathematicians. For although FLT aroused curiosity among number-theorists, the problem remained on the margins of the field for decades. Remarkably few serious efforts were devoted to it during Vandiver's lifetime. Moreover, his research program involved the kind of massive calculations of individual cases that most number-theorists consciously avoided. Aided by electro-mechanical and, later on, electronic devices for making such calculations, Vandiver emerged as a prominent exponent of a research style that many of his contemporaries would have considered unworthy of a true mathematician's time. Nor has his reputation benefited posthumously from the lavish praise that Andrew Wiles's work received. Since Wiles's general proof came from a completely different direction that bore little relationship with Vandiver's train of ideas, the latter's contributions have either been overlooked or are seen today as devoid of direct interest for actual research in number theory.

Figure 2. Maude, Frank and Harry S. Vandiver (HSV).

Nonetheless, from a historical point of view, Vandiver is a figure of considerable interest, not only because of his intense involvement with FLT but also for the role he played within the American mathematical community throughout his long and, in many ways, exotic career. The present article offers a portrait of that career, along with a brief sketch of Vandiver's activities in connection with FLT. In a follow-up to this article I will describe some other aspects of Vandiver's work not directly connected with FLT. An interesting, though somewhat elusive historical question that I will attempt to answer at least partially concerns Vandiver's professional status in the eyes of his contemporaries.

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A self-trained mathematician, Harry Schultz Vandiver was born October 21, 1882, in Philadelphia. He never completed high school, and the little college- and graduate-level mathematics he studied at the University of Pennsylvania in 1904–06 was undertaken in a rather haphazard, non-systematic manner. Thus he never obtained a college degree, except for an honorary doctorate that the University of Pennsylvania bestowed upon him in 1945 at the age of 63.[2]

In 1900 he started submitting solutions to problems posed in the *American Mathematical Monthly*, especially on topics in algebra and number theory.[3] This activity seems to have been his gateway to studying mathematics, and it was certainly how he got to know George David Birkhoff (1884–1944) well before the latter became the most influential American mathematician of his generation. The young Birkhoff wrote to Vandiver in 1901 commenting on the latter's contributions to the *Monthly* while telling him about his own interest in FLT.[4] Thus began a substantial correspondence that lasted several years, though they did not meet until 1913. Their joint paper, in 1904, was Birkhoff's first publication.[5]

Between 1905 and 1917 Vandiver was working as a customs house broker and freight agent for his family's firm. In his letters to Birkhoff—written on stationery with the letterhead of the "John L. Vandiver, Custom House Broker" in Philadelphia—he openly expressed his admiration for his friend's knowledge and "boundless enthusiasm for mathematics." Occasionally he suggested ideas intended for possible additional joint publications, but these never actually materialized. Birkhoff's juvenile interest in number theory and

[1][Corry 2008].

[2]There are various sources of information about Vandiver's life, sometimes containing contradictory information. I have drawn here mainly on documents found at the Vandiver Collection, Archives of American Mathematics, Center for American History, The University of Texas at Austin. See also [Greenwood *et al.*, 1973], [Lehmer 1973].

[3]See *Am. Math. Mo.* 7 (May 1900), p. 146. His first number-theoretical problem appears in the *Am. Math. Mo.* 8 (Aug. 1901), p. 180. A more significant one, dealing with properties of Mersenne numbers appeared in *Am. Math. Mo.* 9 (Feb. 1902), pp. 34–36.

[4][Vandiver 1963, 271].

[5][Birkhoff & Vandiver 1904].

During times of intense research effort, he would isolate himself from all distractions.

elementary geometry soon began to recede in favor of his mature pursuits in analysis and applied mathematics. Vandiver remained strongly focused on number theory and on related algebraic disciplines and consistently tried to pull Birkhoff back into these fields. In 1915 Vandiver wrote to him:

> I am particularly anxious that you become interested in number theory. If I can induce you to take up the subject I am sure you will never regret it. Your position in the math[ematical] world is now assured, and I think you should be able to give considerable time to these things which virtually constituted the life work of such men as Gauss, Kummer, Kronecker, Dirichlet—after your present work is completed.[6]

Birkhoff, in turn, consistently encouraged his friend to pursue his mathematical interests beyond what his free time in business would allow him. Vandiver eventually found himself in a real dilemma, but it took some time before he finally decided to embrace mathematics as a profession. At one point he wrote to Birkhoff, half joking-half serious, that with business slackening because of the war, he now had considerable time for research. "Perhaps times will become so bad," he added, "that I will be compelled to look for some teaching position."[7] From 1917 to 1919 Vandiver served as yeoman in the U.S. Naval Reserve, but soon afterward, aided by Birkhoff's active endorsement, he accepted a teaching position at Cornell.

With his arrival at Cornell, Vandiver began collaborating with Chicago's Leonard Eugene Dickson (1875–1954), who was preparing his monumental *History of the Theory of Numbers*. In particular, Vandiver was actively involved in writing the chapter on FLT, and in 1928 he was co-author of a supplementary volume to Dickson's work.[8] *Algebraic Numbers* was produced on Dickson's recommendation as the official report of the Committee on Algebraic Numbers of the National Research Council, a committee Vandiver chaired between 1923 and 1928. This collaboration with L. E. Dickson left a deep imprint on Vandiver. Throughout the years, he continually referred to the spirit of Dickson's work as an example that should be followed in all of mathematics, and he attempted to implement several initiatives along these lines. Such undertakings included detailed bibliographies of various individual mathematical domains as well as proposals for significant reforms in mathematical reviewing and the refereeing systems in the USA.

Strongly recommended by Dickson, Vandiver was appointed in 1924 to a professorship at the University of Texas, Austin. This became his academic home until the end of his life, though his relationship with colleagues at this institution can hardly be described as one of peaceful coexistence. Above all, his personal and professional relations with the almighty Robert Lee Moore (1882–1974) were a source of constant strain that reached remarkable peaks of mutual animosity.[9] Beyond solitary research, Vandiver's main strengths were clearly not in classroom teaching, but rather in direct and personal interchanges.[10] While his work involved active collaboration with several younger mathematicians, and particularly graduate students, he formally directed only five Ph.D. dissertations at Austin.[11] One cannot help but compare him in this regard with Moore, who devoted a great deal of his energy to advising many promising Ph.D. candidates. Eventually a network of Moore's academic descendants found positions at departments throughout the country, and their efforts helped make point-set topology a leading field of research in the USA. Vandiver's network remained far more circumscribed, reflecting the more limited interest in FLT and other research topics he pursued throughout the years.

Nevertheless, Vandiver traveled extensively and took repeated leaves of absence to pursue his research. Much of his correspondence with university authorities revolved around requests related to these leaves. Thus, it was with a touch of irony that in its sympathetic Memorial Resolution of 1973, Vandiver was remembered by the Faculty Council as a distinguished former UT professor, whose colleagues "bemoaned the fact that he did not *stay*" around very much.[12] He was constantly applying for research grants provided by a number of institutions, including the National Science Foundation, the Carnegie Institution, and the Guggenheim Foundation. In 1934, he was the first mathematician ever to apply for support from the American Philosophical Society.[13] In 1953, at the age of seventy-one, he requested (for the sixth time) funding from the Guggenheim Foundation for a planned six-month leave of absence. Surprisingly the Foundation granted him approval, but then Vandiver decided to withdraw his application.[14] Even at the age of 76 he received a research grant from the NSF. Although those within his close circle of friends would invariably write the warmest letters of recommendation in support of his many applications, this was not always the case with others. Birkhoff, for instance, advised the Guggenheim Foundation that it would be better to devote its resources to support younger men, and he saw no reason why Vandiver could not continue to pursue his research at his home institution.[15] Still, Birkhoff and others con-

[6]Vandiver to Birkhoff: March 18, 1915 (HUG).
[7]Vandiver to Birkhoff: May 17, 1915 (HUG).
[8][Vandiver & Wahlin 1928].
[9]I will deal with this in the follow-up to this article; part of the story is told in [Parker 2005, 226–231].
[10][Lehmer 1973] describes Vandiver as a "poor lecturer." Some of Vandiver's students expressed similar views elsewhere.
[11]They are: Ferdinand Biesele (1941), Olin Faircloth (1951), Charles Nicol (1954), Milo Weaver (1956), and Richard Kelisky (1957).
[12]http://www.utexas.edu/faculty/council/2000-2001/memorials/SCANNED/vandiver.pdf.
[13]Edwin G. Conklin to Vandiver: July 6, 1934 (APS).
[14]Vandiver to Henry Allen Moe: June 17, 1953 (HSV).
[15]Birkhoff to Foundation: March 7, 1953 (GFA).

tinued to stress Vandiver's status as the acknowledged world-leading expert on FLT and to praise his (not totally unrelated) work on cyclotomic fields. In later years, many expressed admiration for the fact that a man of Vandiver's age could still be an active researcher.[16] This admiration was also often conveyed to him directly in personal letters.[17]

Vandiver's frequent travels were part of a somewhat nomadic lifestyle that often took him, his wife Maude (*née* Folmsbee), and their son Frank (1926–2005) around the country and also to Europe. Frank was home-schooled, as experience had hardened Vandiver's strong distrust of public schools.[18] Frank reported that in his childhood the family never had a permanent home nor did they ever own a house. They would move from one apartment to another, renting the home of a colleague on leave while Vandiver was preparing to go on leave himself, then going back to another rented apartment and so on. For many years Vandiver also had a "permanent" room at the Alamo Hotel.[19] During times of intense research effort (and these were not infrequent), he would isolate himself from all distractions by checking into this hotel room. He always kept a suitcase in his office, just in case such an eventuality might arise. Alternatively, he sometimes preferred to lock himself in at home and remain incommunicado for several days.[20] He could single-mindedly concentrate on his work, sometimes forgetting even to eat, thereby bringing himself to the brink of physical collapse.

Two extra-mathematical topics surface fairly often in his letters: classical music and baseball. Vandiver seems to have owned a remarkable collection of records that he managed to carry along with him as he moved from place to place. He especially loved Mozart, and in various letters he mentioned plans to write a mathematician's guide to listening to this genial composer's music. When it came to baseball, he prided himself in taking original views that went against the flow of mainstream opinion. Commenting on a recent game played by his "favorite team," the New York Giants, against their powerhouse rivals, the Yankees, Vandiver noted that the Giants' "present line up includes many wonderful *fielders*," adding that "defense seems to be paid little attention to these days by the public. They prefer to look at cheap homeruns."[21] Above all, what he enjoyed was the excitement of a tight contest, as "a game which ended with a one-sided score was not to his taste."[22]

All kinds of additional oddities were associated with this somewhat legendary figure on (and off) the Texas campus, as reflected in a sympathetic account written by his Austin colleague Robert Greenwood:

> H.S. Vandiver was hardly the athletic type. There is no record of his ever having owned a car, and so he must have walked a lot. Between semesters in the winters the University buildings frequently were heated up to about 40°F. But Professor Vandiver would walk up to campus, go into his office where he had a portable electrical heater with a spherical or parabolic reflecting surface. He would then work away on a mathematical theorem of current interest to him. Usually he would, on these occasions, work in his top coat with the electric heater warming his feet and legs.[23]

Many friendly exchanges of letters among his posthumous papers offer a closer glimpse into his unique personality. Richard Bellman (1920–1984) was a brilliant and versatile mathematician whose fields of interest were much broader and quite different from Vandiver's. In 1959, Bellman created the *Journal of Mathematical Analysis and Applications* and invited Vandiver to join the editorial board. Vandiver initially declined:

> I am now on a basic research grant of the N.S.F., which does not expire for two years, and as I am now 76 years old, I do not feel that I can take on any mathematical work aside from what the Foundation expects me to do, namely, to do research on number theory.[24]

Bellman did not give up, however, and replied immediately in order to assure Vandiver that he would "have the responsibility of looking over the papers of only one mathematician, namely, yourself." And he added:

> As far as your rather young age of 76 is concerned, I distinctly remember that you are the person who insisted that he wanted to live to be 95 and to be shot by a jealous husband. Consequently, if you divided your remaining activities between number theory and these pursuits, I feel we have the best of the bargain.[25]

Vandiver's later years were affected by poor health. Still, he continued to work under a "Modified Service" appointment until the age of 80; only then did he become Professor Emeritus. When he died on January 4, 1973, he was 89 years old.

Vandiver and FLT in Context

Vandiver did not develop new concepts or overarching theories to deal—from completely novel perspectives—with FLT. Rather, his approach was that of a meticulous technician who fully exhausted the unexploited potential of existing theories and refined them further where necessary. At the center of Vandiver's work one finds extensive, highly complex calculations of particular cases, along with innovations aimed at improving existing computational techniques. He was apparently undaunted by even the most demanding computations, in part because he was willing to use a variety of tools (both material and conceptual) to achieve his task.

In order to understand the context of

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[16]André Weil to Foundation: June 1953 (GFA).

[17]Alfred Brauer to Vandiver: Dec. 23, 1958 (HSV). Brauer assured Vandiver that he would support his request for a grant with the NSF, and the latter was indeed granted.

[18]Frank Vandiver later became a distinguished professor of American history and, among other things, Provost of Rice and President of Texas A&M University.

[19]See [Greenwood *et al.*, 1973, 10932].

[20]Frank Vandiver, interview with Ben Fitzpatrick and Albert C. Lewis, June 30, 1999 (MOHP).

[21]Vandiver to W. L. Ayres: August 31, 1951 (HSV).

[22][Greenwood *et al.*, 1973, 10932].

[23]Robert Greenwood, "The Benedict and Porter Years, 1903–1937", unpublished oral interview (March 9, 1988), (MOHP), p. 26.

[24]Vandiver to Bellman: April 20, 1959 (HSV).

[25]Bellman to Vandiver: April 28, 1959 (HSV).

Vandiver's work, a few words must be said both about the status of FLT at the turn of the twentieth century as well as to the standing of research in number-theory in the United States. During the first third of the twentieth century, the American community of number-theorists was relatively small and not extremely prominent. As a rough measure, one might note that the index to the first ten issues of the *Transactions of the AMS* (1909) lists only two articles under the heading of number theory, and in the following decade, despite a noticeable increase, there were still only thirteen. In his correspondence, Vandiver often spoke about the lack of interest in number theory in the US. Looking through this correspondence for names of mathematicians actively involved in research in number theory prior to 1940, one finds above all foreign figures, some of whom Vandiver also visited in Europe: Rudolf Fueter, Edmund Landau, Emmy Noether, Helmut Hasse, Kurt Hensel, Philipp Furtwängler, Nikolai Grigorevich Chebotarev, Dmitry Mirimanoff, Taro Morishima, Trygve Nagell, Felix Pollaczek, and Arnold Walfisz. We also find a number of American mathematicians, mainly active on the West Coast: Eric Temple Bell, Hans Frederik Blichfeldt, Derrick Norman Lehmer (and later on Derrick Henry and Emma Lehmer), Robert Daniel Carmichael, Albert Cooper, Leonard Eugene Dickson, Morgan Ward, and Aubrey Kempner. Unlike the case with their foreign counterparts, this latter group essentially exhausts that of number-theorists active in the American community. After 1940 the field became more active, though it continued to remain somewhat on the margins of the research community for some years.[26] Vandiver's almost exclusive focus on this field, along with the unusual circumstances surrounding his early mathematical career, helps to account for his rather unique situation within the American mathematical community. His decision to devote so much of his professional life to FLT made him a truly singular figure.

Although even a mini-history of Fermat's problem is well beyond the scope of this article, a bit of historical background is necessary in order to understand Vandiver's work. This sketch will also underscore the fact that rather few significant contributions to solving the problem were made following Ernst Eduard Kummer's work in the 1850s. As part of his long-standing efforts to deal with questions related to higher reciprocity laws, Kummer developed many important concepts and techniques that turned out also to be relevant for FLT. Thus, it was Kummer who introduced the notions of regular and irregular primes, a distinction based on a property of the "class number" h_p of a cyclotomic field $k(\zeta)$. He then developed original methods that enabled him to prove that FLT is valid for all regular primes. In addition, he introduced three somewhat complex conditions which, when satisfied by an irregular exponent p, implied the validity of FLT for that exponent. Then, by means of long and tedious computations, he identified all irregular primes under 164, obtaining these eight numbers: 37, 59, 67, 101, 103, 131, 149, and 157. Applying his criteria to the three cases of irregular primes under 100 (37, 49, 67), he achieved his well-known result of 1857 that FLT is valid for all exponents less than 100. Beyond this, however, the calculations became prohibitively complex and, in addition, it became clear to Kummer that some of the criteria he had developed would not apply for certain irregular prime exponents, such as $p = 157$.

Kummer also proved that a prime p is regular if and only if it does not divide the numerators of any of the Bernoulli numbers $B_0, B_2, \ldots B_{p-3}$, which appear as coefficients in the expansion

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!}.$$

At that time values for the Bernoulli numbers had been calculated up to B_{62}.[27] The ability to identify higher values of regular or irregular primes would later come to depend on the possibility of calculating higher values of these numbers, an effort in which Vandiver was directly involved.

Kummer was both an avid calculator and a gifted theorist, but it was the latter aspect of his work that most influenced the development of number theory in the decades to come, especially through the efforts of Richard Dedekind. By the turn of the twentieth century, particularly in the wake of Hilbert's influential *Zahlbericht*, the emphasis on a "conceptual" perspective (as opposed to the more algorithmic approach favored by Leopold Kronecker) became dominant in the discipline. Results based on special calculations for particular cases were not favored under this view. These general trends in research account to a large extent for the remarkable fact that Kummer's results relating to FLT were not essentially improved or extended for almost sixty years.

Prior to Kummer, Sophie Germain had proved an important result, as a consequence of which the proof of Fermat's problem can be reduced to dealing with two separate special cases. Case I asserts that for $p > 2$, $x^p + y^p + z^p = 0$ has no integer solutions for x, y, z relatively prime to the odd prime p. Case II asserts the same when one and only one of the three numbers x, y, z is divisible by p. The little progress that did take place in proving the theorem between Kummer and Vandiver dealt almost exclusively with case I. The most important single result was obtained in 1909 by Arthur Wieferich (1884–1954). Wieferich proved that if three integers x, y, z relatively prime to p actually did satisfy $x^p + y^p = z^p$, then the congruence $2^{p-1} \equiv 1 \pmod{p^2}$ must be satisfied. Dimitry Mirimanoff (1861–1945) extended this result in 1910 by proving that the same p would also satisfy $3^{p-1} \equiv 1 \pmod{p^2}$. These two results, and some similar ones that were sporadically added later on, helped establish a lower bound for the value of the integers for which the Diophantine equation $x^p + y^p = z^p$ could be satisfied under the conditions of case I (and this, moreover, only by considering p, and irrespective of the values of x,y,z that may satisfy the equation).

Based on these results, and directly motivated by the additional encouragement provided by the creation in 1908 of the Wolfskehl Prize, several mathematicians decided to attack the problem

[26]A detailed analysis of the internal structure of this community and its development (along the lines of [Goldstein 1994] for the case of the French community of number-theorists in the second half of the nineteenth century) seems to be an interesting open task for historical research.
[27][Ohm 1840].

anew, producing several additional results along the same line of ideas. Thus one finds contributions by such leading figures as Philipp Furtwängler and Georg Ferdinand Frobenius, but also by the then unknown Vandiver. In 1914, in his first article on FLT,[28] he proved a Wieferich-like congruence for 5^{p-1}.

Vandiver's first truly substantial result came in 1920, when he identified a mistake in Kummer's article of 1857 and went on to correct one of its main arguments.[29] He continued to refine and develop his ideas on FLT over the next few years. A summary of his achievements appeared in his authoritative article of 1929,[30] for which he was awarded the AMS's first Cole Prize in number theory two years later. This award, honoring the AMS's long-time secretary Frank Nelson Cole, was established for outstanding work in this field.

Vandiver's Contributions to FLT

Vandiver's article of 1921 went well beyond Kummer's results by proving FLT for exponents up to $p = 211$. Even before it appeared in print, Vandiver realized that his arguments could be used to extend the results to $p < 269$. Under his direction, specific calculations of various ranges were performed separately by various M.A. students at Austin, including Samuel Wilks (1906–1964) and Elizabeth Stafford (1902–2002). All calculations of values p, $100 < p < 211$, were performed by Wilks "using Monroe and Marchant electrical computing machines." Vandiver and his team proved that if p divides only one of the numbers B_2, B_4, . . . , B_{p-3}, and if this single Bernoulli number is not divisible by p^3, then FLT is valid for p. This allowed further calculations for exponents $p < 307$. As the difficulty of the calculations increased, Vandiver devised further methods to simplify and speed up the procedures, and also to allow for double-checking.

At the same time, however, he was skeptical about the general validity of Case II of FLT. Eric Temple Bell wrote to him in 1929,

> If I remember rightly, you once said that you would not be surprised if the second case turned out to be false. . . . You give the limit five hundred for exponents to be tried. I have no idea of the actual amount of computation required for such an undertaking, but I should think it would be terrific. There is no doubt in my mind that anyone who knows anything about the Theory of Numbers would say that this work ought to be done while there is a man not only able to do it, but also willing. If in one of these exponents the computations should give a negative result, you will set a problem to exasperate generations of arithmeticians. I rather hope that it does turn out that way.[31]

In the next few years, however, Vandiver would surpass the exponent 500 and would continue to confirm the validity of FLT for ever higher values of p, including those covered by case II.[32]

In an article in 1934 he remarked that much of his "work concerning FLT is tending toward the possible conclusion that if the second factor of the class number" h_p of $K(\zeta)$ is prime to p, "then FLT is true." This is the famous "Vandiver conjecture," about which he had begun to speculate much earlier. Its importance for algebraic number theory in general gradually gained recognition over the years, albeit in somewhat modified versions.[33] This was by no means the only original conjecture that appeared in his articles, however, as was pointed out in later research.[34]

The most significant progress in calculations related to FLT resulted from Vandiver's work with the couple Derrick Henry Lehmer (1905–1991) and Emma Lehmer (1906–2007). Their collaboration started in 1932, though the first joint publication did not appear until 1939, when they proved the validity of FLT for $2 < p < 619$.[35] Above 619, the calculations became prohibitively long and laborious to be carried out with the kind of desktop calculators available to the Lehmers. But in 1953 when electronic computers became available, the three mathematicians took a great leap forward, proving that FLT was true for all exponents $p < 2000$.[36] Throughout their correspondence, Vandiver stressed that, beyond the specific results obtained, this research had an enormous value for advancing research on cyclotomic fields. In subsequent papers, he further refined the Kummer criteria for irregular primes, and this led to an extension of the results on FLT to $2000 < p < 2520$ in 1954 and then, in 1955, to p in the range $2520 < p < 4002$.[37]

This work of Vandiver and his collaborators using electronic computers did not, however, alter mainstream research in number theory, at least not in the short run. Nor did it lead to renewed research efforts in connection with FLT. Still, seen in retrospect, these pioneering efforts opened the door to a new direction of research that remains active today. Additional results along similar lines have continued to confirm FLT for exponents exceeding one billion, and in Case I for higher values still. In fact, even after Wiles's general proof of FLT, new ranges of exponents are still being tested with ever improved techniques.[38]

In 1946, following a request from the editors of the *American Mathematical Monthly*,[39] Vandiver published a de-

[28][Vandiver 1914].
[29][Vandiver 1920, 1922].
[30][Vandiver 1929].
[31]Bell to Vandiver: Jan 15, 1929 (HSV).
[32]For a detailed account of Vandiver's works during these years and how they eventually led to the use of electronic computers to solve FLT, see [Corry 2007].
[33]See, for instance, [Iwasawa & Sims 1965]. [Lang 1978, 142] pointed out that the conjecture had originally been formulated by Kummer [*Coll.* Vol. 1, 85]. Lang indicated that "Vandiver never came out in print with the statement: "I conjecture etc. . . . ", but "the terminology 'Vandiver conjecture' seemed appropriate to me. In any case I believe it".
[34][Herstein 1950, Dénes 1952].
[35][Vandiver 1937, 1937a].
[36][Vandiver, Lehmer & Lehmer 1954].
[37][Vandiver 1954, Vandiver, Selfridge & Nicol 1955].
[38][Wagstaff, 1978; Buhler *et al.* 1992; Buhler *et al.* 2001].
[39]Lester R. Ford to Vandiver: February 2, 1945 (HSV).

tailed exposition of the state of the art in research on Fermat's problem.[40] This article became a classical locus of reference for many years to come. In summarizing his opinion about the general validity of the conjecture, about which he was frequently asked, Vandiver drew a clear distinction between the two classic cases. He was convinced of the validity of case I, but not merely because it had been proved for very high values. Rather, his confidence in this case stemmed from some important theorems he had proved along the way on trinomial congruences—a topic to which he had devoted many efforts. Case II involved a much more complex situation; thus, while he believed it would ultimately be proven, he did not think he had any compelling evidence to support it. Furthermore, he stated, he felt less sure than in 1934 about the validity of the Vandiver conjecture, precisely because of its close relationship with and possible dependence on the validity of FLT. Commenting on the frequency with which apparently promising conjectures in number theory are eventually abandoned, he added:

> When I visited Furtwängler in Vienna in 1928 he mentioned that he had conjectured the same thing before I had brought up any such topic with him. As he had probably more experience with algebraic numbers than any mathematician of his generation, I felt a little more confident. (p. 576)

While mentioning some additional results presented in his current account, he echoed the opinion voiced several years earlier in Bell's letter:

> However it would probably be best if I were wrong about this. I can think of nothing more interesting from the standpoint of the development of number theory, than to have it turn out the Fermat relation has solutions, for a finite number > 0, of primes l.

Concluding, he wrote,

> Many mathematicians are often interested in ascertaining how a particular topic connects up with other parts of mathematics. In case of Fermat's Last Theorem it is well known that Kummer's attempts to prove it gave rise to the theory of ideals which is now of fundamental importance in many parts of mathematics. The remarkable character of Kummer's achievement has tended, however, to minimize the great number of connections which the theorem has with other subjects. Efforts on my part to clear up the question have led me into the following topics: Bernoulli numbers and polynomials and generalizations; Euler and Gennochi numbers; Euler and Mirimanoff polynomials; partitions modulo m; finite fields and rings, including a great many types of congruences: the Dirichlet Zeta Function and the related Dedekind Function; the Lagrange resolvent and Jacobi ϕ number and various generalizations including generalized Gauss sums; the theory of Kummer fields, class number, class fields, power characters and laws of reciprocity in the theory of algebraic fields; Fermat's quotients and other arithmetic quotient forms; congruence theories as applied to power series; abstract algebra including, particularly, group theory and semi-groups; and many types of Diophantine equations aside from the Fermat relation itself.

It seems Vandiver was rather carried away with enthusiasm. Only some of these topics have substantial, direct connections with FLT. On the other hand, Vandiver himself was led to explore many of these potential payoffs, partly because of his interest in questions that arose from, or were thought to be useful for solving FLT. In fact, in 1952–53 he published a two-part article on associative algebras and the algebraic theory of numbers, a paper he regarded as more important and innovative than any of his work directly connected with FLT. He was disappointed that this article was seldom cited:

> As far as I know, only one person has studied thoroughly this paper, and he is Alonzo Church.[41]

In private correspondence Church raised some interesting criticisms regarding the axiomatic debate developed by Vandiver, and Vandiver was quick to include Church's comments in a follow-up to this article.

As late as the early 1960s Vandiver was still publishing new results related to FLT. He also continued to work on a book about FLT and related topics in number theory, a project he pursued for many years. His archive contains hundreds of typewritten pages with whole chapters nearly ready for publication, but for some reason this book was never published.

Correspondence on FLT

As Vandiver came to know, having your name publicly attached to Fermat's problem could impose a considerable burden on a mathematician. Yet some experts found efficient ways to duck the unwelcome task of reading the steady stream of faulty proofs submitted by rank amateurs. In the early twentieth century, Edmund Landau came up with a nearly ideal solution to this corollary to Fermat's problem. As Göttingen's leading authority on number theory, Landau was officially entrusted with handling all correspondence related to the Wolfskehl Prize, which offered 100,000 Marks to anyone who could solve Fermat's conjecture. Landau had little interest in the problem, so he took on this duty with little enthusiasm. When the flow of incoming correspondence from amateurs eventually became unbearable, he became openly disgusted. So he prepared a form letter that looked something like this:

> Dear . . . . ,
>
> Thank you for your manuscript on the proof of Fermat's Last Theorem.
>
> The first mistake is on:
>
> Page . . . . . . Line . . . . . .
>
> This invalidates the proof.
>
> Professor E. M. Landau

An assistant read through the manuscripts and filled in the missing details in the form letter.

Fortunately for Vandiver, no rich oilman came along to establish a similar prize fund at the University of Texas for a successful proof of FLT. So this circumstance surely diminished the number of would-be problem solvers who might have written to him. Nevertheless he did receive enormous quantities of mail that only grew from year to year.

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[40][Vandiver 1946].

[41]Vandiver to Mientka: March 13, 1964 (HSV). See also, Vandiver to Herstein, April 2, 1960.

In fact, many American mathematicians (perhaps all of them?) saw in Vandiver the default address to which any letter on the topic should be redirected. Vandiver's attitude toward these correspondents was essentially positive, perhaps because he had been something of an amateur mathematician himself. In response to attempted proofs by rank amateurs, he sent a pre-written, but rather polite reply. His archives contain no fewer than 225 such answers, sent between 1934 and 1966. To those he considered qualified mathematicians he usually answered in some detail, though even in these cases the task became increasingly onerous with time.

An interesting letter from 1949 attests to this problem in the case of a mathematician, Taro Morishima, whose contribution Vandiver truly appreciated. He was forewarned that Morishima was about to submit an article on FLT to an American journal. This prompted him to take preemptive action by contacting several editors (Aurel Wintner of the *American Journal of Mathematics*, Rudolph E. Langer, Saunders Mac Lane, and Leonard Carlitz) to request that the article not be sent to him. He would certainly like to read this paper, he said, but at his leisure and not under pressure to finish within some given period of time, however reasonable. Number theory, he added, "seems to be getting popular," but Vandiver felt he was drowning under the enormous correspondence he now had to handle.[42] A few months later, he again complained bitterly about this to another colleague, while requesting that no further letters be sent to him. Only if he received a manuscript from Siegel, Hasse, or Rademacher would he be willing to examine the work in detail. To which he added: "After nearly forty years of looking at such manuscripts, good and bad, don't I deserve a respite?"[43]

One revealing interchange took place in 1961 around a proposed proof of FLT by Lucien Hibbert, then Executive Director of the Inter-American Bank for Development in Washington, D.C. Hibbert had been directed to Vandiver by Israel Herstein and Marshall Stone. Born in 1899 in Haiti, he had received in 1937 a Ph.D. at the Sorbonne, working with Arnaud Denjoy (1884–1974). In Haiti he had been professor of mathematics and physics, and director of the Haitian Statistical Institute, and had enjoyed a very successful career in the finance sector. He was Ambassadorial Representative of Haiti to the Organization of American States, and later served in the Ministry of External Affairs. As usual, Vandiver read the manuscript fairly carefully and replied politely and in some detail. In his answer, he summarized his general attitude toward this matter:

> Next October 21 I shall start my 80th year of age. Beginning in the year 1914 I published several articles on the Fermat problem which received attention from readers to the extent that many of them wrote letters to me, generally containing their opinions . . . about the problem, and also what they regarded as proof of Fermat's statement or contributions to that end. For some years I made a practice of replying to such letters and giving my estimates of the value of their work. However, as I continued to publish from time to time through the years articles pertaining to the Fermat problem, my correspondence along that line became so heavy that if I had continued to do this it would have taken <u>most</u> of my time. . . . As an example of this, I have received four letters within the last few weeks and about seven since the first of the year pertaining to the Fermat problem . . . one of them . . . said the full proof of the theorem covered about 50 pages. In his resume of the paper given me in his letter, he made a number of statements I could not understand at all; . . . so I told him I was sorry I had to refuse to help him. . . .
>
> In my 50 years' experience with the problem I have often been convinced for a time that I had a proof of the theorem using only the tools of elementary number theory and algebra, but I found *in every such case* that I had an error in my argument. After a time I became more and more skeptical of any apparent proof that I found using such elementary means, as I felt that if an elementary proof existed, it could hardly have escaped the attention of such great mathematicians as Euler, Legendre, Lamé, Abel, Gauss, Cauchy, and Kummer, all of whom worked at the problem! . . .
>
> I have looked over the general character of your argument . . . and as far as I can see . . . you have used nothing but elementary algebra therein, hence I cannot help being skeptical as to the accuracy of your work. *If you regard this as a disparagement of your work, please note that I have just disparaged above all my own efforts of this character.*[44]

Vandiver further advised Hibbert to write up full proofs for various specific cases, to see if they worked. And he added: "In giving you this advice, I am assuming that you would prefer to find the error yourself, if one exists, than to have someone else find it." Then, in a letter to Stone he explained what he really feared about cases like this one:

> For many years, in connection with "proofs" of FLT sent to me and which I examined it turned out in nearly every case that if I called the author's attention to an error in his work, soon after I would receive another ms. which he assumed was a correction of his original paper. Also, if instead of pointing out an error I would merely state to him that there was a step in his argument which I did not understand, then the author would reply that I did not understand his [entire] argument. These things would be the beginning of a long correspondence that I would have with him.[45]

Some years prior to Hibbert, Vandiver was involved in another noteworthy exchange with a Pakistani air force officer named Quazi Abdul Moktader Mohd Yahya, who was formerly "Professor of Mathematics at Brajali Academy, East Pakistan." In various letters

[42]Vandiver to various: December 8, 1949 (HSV).

[43]Vandiver to J. R. Kline: January 12, 1950 (HSV).

[44]Vandiver to Hibbert: June 23, 1961 (HSV). Emphasis in the original.

[45]Vandiver to Stone: June 26, 1961 (HSV).

written to colleagues about this man, Vandiver referred to him as X, noting that "I do not wish to be sued for libel, in case the information in this letter somehow reaches him." He received a manuscript from him on FLT that the author wished to submit to the *Proceedings of the National Academy of Sciences*. As in other cases, Vandiver answered the initial letter politely, but this led to a lengthy and futile series of interchanges. Vandiver tried to put an end to this by suggesting that Yahya send his manuscript to a "regular mathematical journal," one "preferably in Switzerland or Germany where they seem to have more interest in number theory than in the U.S." He feared that this "dangerous character" might "write me a threatening letter, as some of these birds have done in the past."[46] Eventually Yahya was able to publish his (obviously flawed) proof in a Portuguese journal in 1976.[47]

A third interesting correspondence over FLT took place in 1960–61 when Vandiver was contacted by a junior high school pupil from Baltimore named Joel Weiss. After learning the names of the three persons in the US who had recently done work on the Fermat problem, Joel wrote to Vandiver (and also to the Lehmers) for advice on this topic, which he had chosen for a school term paper. He was willing to work hard, and so he explained his choice as follows:

> This theorem, which originally was a curiosity to me, turned out to be a stimulating research project well worth the 45 hours of work necessary to complete it. I hope that my conclusion will start a new train of thought leading to an eventual proof of Fermat's Last Theorem.

Joel later indicated at the end of his finished paper what this desired train of thought might be:

> I conclude that Fermat's Last Theorem has been proven all this time, and that its entire proof is that of $n = 3$. I have reached this conclusion from an analyzation of a succession of cases of the theorem with exponents 3 through 9. After studying these cases, it is apparent that the deviation between the sum of the terms in the left-hand member of the equation and that of the right-hand member increases steadily with higher exponent value. Therefore, I feel that it is only necessary to prove $n = 3$ because this is the point of lowest deviation. Any exponent value above this is immediately ruled out as a result of the fact that the deviation is greater than that of the third power thus making it impossible to suit the equation.[48]

Vandiver, who had written several polite and possibly helpful letters to Joel along the way, also reacted politely to Joel's conclusion: most mathematicians, he kindly remarked, would not agree with the closing statement of his paper.

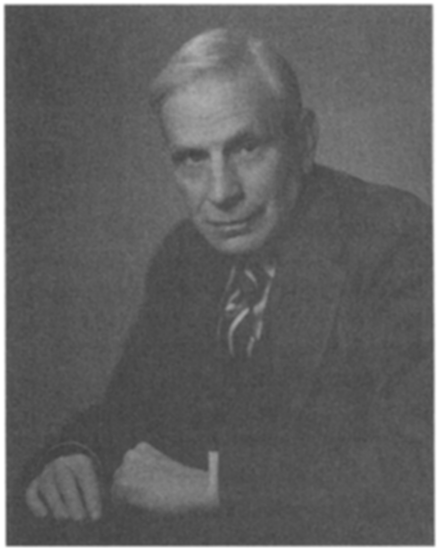

Figure 3. Harry S. Vandiver (Creator: Walter Barnes Studio (HSV).

Recognition and Oblivion

In the currently available literature, Vandiver's name is barely mentioned in connection with FLT. For instance, in the popular "MacTutor History of Mathematics Archive" website, Vandiver barely rates a very short entry of his own. His name appears only in passing in the site's article on FLT, and he is not mentioned at all in the article on Dick Lehmer. From the point of view of *current mathematical research* associated with the problem, especially following Wiles's dramatic breakthrough, this may be understandable. But from the point of view of the *history of the problem*, this lack of recognition is completely unjustified, though the reasons for this are not difficult to find.

Although Vandiver was the undisputed world's leading expert on FLT during his lifetime, contemporaries often took an ambivalent attitude toward him and his passionate quest. Certainly he was well-known and respected both within the American mathematical community and abroad, but his interests were also viewed as exotic, and evidence abounds that he was viewed as more bizarre than brilliantly original. Thus, it is not surprising that when his friend G. D. Birkhoff prepared a list of the 10 most prominent American mathematicians in 1926 for the Rockefeller foundation, Vandiver, then 44 years old, was not on his list.[49] Even during his most creative phase as a researcher he seems to have received less recognition than he probably deserved.

Yet Vandiver received several high honors, including the Cole Prize and an honorary doctorate from the University of Pennsylvania; and, of course, he was the recipient of many research grants. Harry Vandiver was the only American mathematician whose work received mention in Edmund Landau's 1927 classic textbook on number theory. He was elected vice-president of the AMS for the term 1933–1935, and in 1935 he was an AMS Colloquium Lecturer. He served as assistant editor of the *Annals of Mathematics* from 1926 to 1939, and in 1934 he was elected to the National Academy of Sciences. Still, he always remained part of a small and rather marginal sub-community within the larger American mathematical research enterprise. Strongly fixated on his own work, he was certainly not a shaker and mover. He would not manage to attract large numbers of young researchers to his chosen field; he did not establish a research school, nor did he develop an influential network of contacts with like-

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[46]Vandiver to Hayman: April 3, 1958 (HSV).

[47]*Mathematical Reviews* lists a "private edition" by the author [Yahya 1958], and three additional articles in *Portugaliae Mathematica* (1973, 1976 and 1977).

[48]The entire correspondence appears in HSV: File 16-3.

[49]See [Siegmund-Schultze 2001, 51]. Birkhoff's list included only mathematicians from three leading centers: Cambridge (Birkhoff, Morse, Osgood, Wiener, Whitehead); Chicago (Bliss, Dickson, E. H. Moore, Moulton); and Princeton (Alexander, Eisenhart, Lefschetz, Veblen).

minded mathematicians. Nor was he an organizational talent who excelled when it came to promoting journals or organizing professional meetings.

The honors conferred on Vandiver occasionally betray ambivalence. For example, only after Vandiver himself applied some direct pressure on university authorities was he named Distinguished Professor at TU in Austin, in 1947. But his title, "Distinguished Professor of Applied Mathematics and Astronomy," was certainly odd given his research expertise.

More telling still is the context surrounding a *Festschrift* published in his honor. In 1966 Bellman's *Journal of Mathematical Analysis and Applications* brought forth the special issue dedicated to Vandiver on his eighty-third birthday. The editors wished to honor him not only for his contributions to FLT and algebraic number theory but also because "he has profoundly influenced the development of American mathematics for a period of over sixty years." And yet the American contributions to this volume were all written by his former students and close collaborators. Side by side with these papers one finds a score of others written by leading number-theorists from abroad, figures such as Mordell, Hasse, Erdős, Szemeredi, Gel'fond and Morishima. It's odd that such a collection appeared in a journal far removed from Vandiver's own fields of interest. Evidently the decision to publish such a *Festschrift* came from close friends who wanted to pay long-overdue tribute to the man and his work, yet sensed that no one outside Vandiver's inner circle would ever undertake it. The honoree, then in delicate health after undergoing surgery, was deeply touched by this gesture.[50]

Vandiver's lifetime endeavor was characterized by remarkable independence and a willingness to pursue self-styled, original research programs. As a researcher, his style was marked by an indefatigable appetite for endless calculations, by a peculiar style of collaboration with small groups of people who were close to him, and by his pioneering use of electronic computers in his fields of expertise. While Vandiver's contributions played no direct role in shaping the train of ideas that eventually led to the general proof of FLT, and while opinions may vary as to the intrinsic mathematical significance of the ideas developed in his work, one cannot make sense of the *history* of FLT without giving prominence to the story of this man, the only one ever to devote his entire professional life to solving the problem.

Figure 4. AMS-MAA meeting in Washington D.C. (HSV). Source: Capitol Photo Services, Inc.

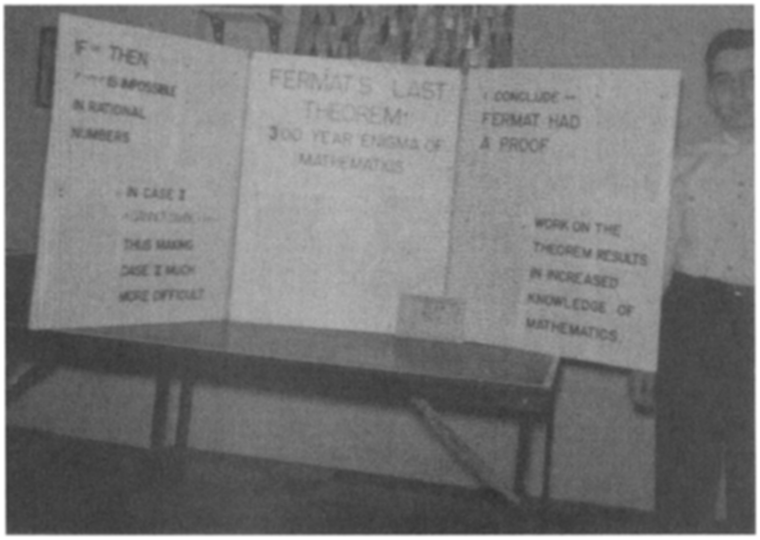

Figure 5. Joel Weiss with a poster presentation of his work on FLT (HSV).

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I have used archival material found in several institutions. I thank the archivists for assistance in locating and copying the originals, and for granting permission to quote. Pictures are reproduced and sources are quoted with permission, using the following abbreviations:

HSV: Vandiver Collection, Archives of American Mathematics, Center for American History, The University of Texas at Austin.

MOHP: Oral History Project, The Legacy of R.L. Moore, Archives of American Mathematics, Center for American History, The University of Texas at Austin.

HUG: George David Birkhoff Papers, Harvard University Archives: Call Number HUG 4213.2, Box 3, Folder "T-V".

APS: American Philosophical Society Archive.

GFA: The John Simon Guggenheim Memorial Foundation Archive.

[50]Dorothy W. Baker to Bellman: September 29, 1965 (HSV).

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